Decays of $\tau \to \rho(770)(\rho'(1450))\nu_\tau$ and $\tau \to K^*(892)(K^{*'}(1410))\nu_\tau$ in the extended Nambu - Jona- Lasinio model

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Abstract

In the extended Nambu - Jona - Lasinio model the decay widths $\tau \to \rho(770)(\rho'(1450))\nu_\tau$ and $\tau \to K^*(892)(K^{*'}(1410))\nu_\tau$ are studied in the quark one -loop approximation. Our estimations of the decay widths $\tau \to K^*(892)(K^{*'}(1410))\nu_\tau$ are in satisfactory agreement with experimantal data. In the paper, the decay widths $\tau \to \rho(770)(\rho'(1450))\nu_\tau$ are also calculated.

Keywords: Nambu - Jona - Lasinio model, excited mesons, τ decays

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I. INTRODUCTION

Recently, in the framework of the extended Nambu - Jona - Lasinio (NJL) model [1–4] a number of processes connected with the creation of mesons in τ decays and in the reaction of $e^+e^- \to hh$ at low energy were successfully described.

Such processes are $\tau \to \pi^-\pi^0\nu_\tau$ [5], $\tau \to \eta(\eta')\pi^-\nu_\tau$ [6], $\tau \to \eta(550)(\eta'(950))2\pi\nu_\tau$ [7], $\tau \to \pi^-\omega\nu_\tau$ [8]. In these reactions it is necessary the take into account in the intermediate states both the ground state $\rho(770)$ and the first radial excited state $\rho'(1450)$.

A similar mechanism can be used for the description of the reactions $e^+e^- \to hh$ at low energy. Here the intermediate ρ^0, ω, ϕ mesons and their first radial excited states are used. These processes are $e^+e^- \to \pi^0(\pi^{0'})\gamma$ [9], $e^+e^- \to (\eta(550), \eta'(950), \eta(1295), \eta(1475))\gamma$ [10], $e^+e^- \to \pi\pi(\pi'(1300))$ [11], $e^+e^- \to \pi^0\omega$ [12], $e^+e^- \to \pi^0\rho^0$ [13], $e^+e^- \to \eta(550)(\eta'(950))2\pi$ [7].

Naturally it is interesting to describe the decays $\tau \to \rho(770)(\rho'(1450))\nu_\tau$ which are the basis of the above-mentioned processes. This paper is devoted to the solution of this problem. Also, it is very interesting to consider the decays $\tau \to K^*(892)(K^{*'}(1410))\nu_\tau$ as there are reliable experimental data for them [14]. It is shown that our results obtained in the framework of the extended NJL model are in satisfactory agreement with these experimental data.

II. LAGRANGIAN OF THE QUARK - MESON INTERACTIONS IN THE EXTENDED NAMBU - JONA -LASINIO MODEL

The Lagrangian of the quark - vector meson interactions in the extended Nambu - Jona -Lasinio model has the following form:

$$\begin{aligned}\Delta\mathcal{L}^{int} &= \bar{q}(k')\big[i\hat{\partial} - m + A_\rho\lambda_3\gamma_\mu\rho_\mu(p) - A_{\rho'}\lambda_3\gamma_\mu\rho'_\mu(p) \\ &\quad + A_{K^\star}\lambda_\pm\gamma_\mu K^\star_\mu(p) - A_{K^{\star\prime}}\lambda_\pm\gamma_\mu K^{\star\prime}_\mu(p)\big]q(k), \qquad (1)\end{aligned}$$

where $\hat{\partial} = \gamma_\mu\partial_\mu$, $m = \mathrm{diag}(m_u, m_d, m_s)$, $m_u = m_d = 280$ MeV, $m_s = 405$ MeV, q and $\bar{q}$ are the quark fields, $\rho_\mu(\rho'_\mu)$ and $K^*_\mu(K^{*'}_\mu)$ are the vector meson fields in the ground (excited)

state

$$
\begin{aligned}
A_{\rho} &= g_{\rho_1}\frac{\sin(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k_\perp^2)\frac{\sin(\beta-\beta_0)}{\sin(2\beta_0)}, \\
A_{\rho'} &= g_{\rho_1}\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k_\perp^2)\frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)}, \\
A_{K^\star} &= g_{K^*}\frac{\cos(\theta+\theta_0)}{\sin(2\theta_0)} + g_{K^{*\prime}} f(k_\perp^2)\frac{\cos(\theta-\theta_0)}{\sin(2\theta_0)}, \\
A_{K^{\star\prime}} &= -g_{K^*}\frac{\sin(\theta+\theta_0)}{\sin(2\theta_0)} - g_{K^{*\prime}} f(k_\perp^2)\frac{\sin(\theta-\theta_0)}{\sin(2\theta_0)},
\end{aligned} \tag{2}
$$

$$
\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_+ = \sqrt{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_- = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
$$

The values of the angles $\beta = 79.85^\circ$ and $\beta_0 = 61.44^\circ$ are taken from [2], and $\theta = 84.7^\circ$, $\theta_0 = 59.14^\circ$ [4] are the mixing angles for the ground and first radially excited states of mesons, respectively.

Radially excited states are described in the extended NJL model using the following form factors $f(k_\perp^2)$ in the quark-meson interaction:

$$
\begin{gathered}
f(k_\perp^2) = (1 - d|k_\perp^2|)\Theta(\Lambda_3^2 - |k_\perp^2|), \\
k_\perp = k - \frac{(kp)p}{p^2}, \quad d = -1.784 \text{ GeV}^{-2},
\end{gathered} \tag{3}
$$

where k and p are the quark and meson momenta, respectively, and the cut-off parameter $\Lambda_3 = 1.03$ GeV. The quark-meson coupling constants are

$$
\begin{aligned}
g_{\rho_2} &= \left(\frac{2}{3} I_2^{f^2}(m_u, m_d)\right)^{-1/2} = 9.87, \qquad g_{\rho_1} = \left(\frac{2}{3} I_2^{(0)}(m_u, m_d)\right)^{-1/2} = 6.14, \\
g_{K^{*\prime}} &= \left(\frac{2}{3} I_2^{(f^2)}(m_u, m_s)\right)^{-1/2} = 10.86, \qquad g_{K^*} = \left(\frac{2}{3} I_2^{(0)}(m_u, m_s)\right)^{-1/2} = 6.77,
\end{aligned} \tag{4}
$$

where the integrals $I_m^{f^n}$ read

$$
I_m^{f^n}(m_q) = -i\frac{N_c}{(2\pi)^4}\int \mathrm{d}^4k \frac{(f_q(k_\perp{}^2))^n}{(m_q^2 - k^2)^m}\Theta(\Lambda_3^2 - k_\perp^2), \tag{5}
$$

where $N_c = 3$ is the number of color.

III. AMPLITUDES OF THE DECAYS $\tau \to V(V')\nu_\tau$ IN THE EXTENDED NJL MODEL

The Feynman diagram for the decay $\tau \to \rho(\rho')\nu_\tau$ is shown on Fig. 1. The amplitude of this decay has the form

$$A_{\tau\to\rho(\rho')\nu_\tau} = \frac{G_F}{\sqrt{2}} \cdot \bar{u}_{\nu_\tau}\gamma_\alpha u_\tau \cdot g_{\alpha\mu} \cdot |V_{ud}| \frac{g_\rho}{2} \int \frac{d^4k}{(2\pi)^4} \frac{tr\ [\gamma_\mu((\hat{k}+\hat{p})+m_u)\gamma_\nu(\hat{k}+m_u]e^\nu_\rho(p_\rho)}{(k^2-m_u^2)((k+p)^2-m_u^2)}. \quad (6)$$

Here p is the ρ - meson momentum, $G_F = 1.16637 \cdot 10^{-11} MeV^{-2}$ - is the Fermi constant, k is the quark momentum, m_u - is the u - quark mass, and $|V_{ud}|$=0.97428 is the Cabibbo - Kobayashi - Maskawa mixing angle.

The square of the amplitude takes the form

$$|M|^2 = 4m_\tau m^2_{\rho(\rho')} E_\nu |V_{ud}|^2 \frac{G_F^2}{2} \frac{1}{g^2_{\rho(\rho')}} \left[2E^2_{\rho(\rho')} + m^2_{\rho(\rho')} - 2E_{\rho(\rho')}\sqrt{E^2_{\rho(\rho')} - m^2_{\rho(\rho')}}\right] \quad (7)$$

The decay width for the process is

$$\Gamma(\tau \to \rho(\rho')\nu_\tau) = \frac{|M|^2}{2\cdot 2m_\tau}\Phi, \quad (8)$$

where Φ is the phase volume:

$$\Phi = \frac{E_\nu}{4\pi m_\tau}, \quad (9)$$

and E_ν and E_ρ are determined as

$$E_\nu = \frac{m_\tau^2 - m^2_{\rho(\rho')}}{2m_\tau}, \qquad E_\rho = \frac{m_\tau^2 + m^2_{\rho(\rho')}}{2m_\tau}, \quad (10)$$

We also use $(p_\nu p_\tau) = m_\tau E_\nu, \quad (p_\tau p_{\rho(\rho')}) = m_\tau E_{\rho(\rho')}$.

The Feynman diagram for the decay $\tau \to K^\star(K^{\star'})\nu_\tau$ is shown on Fig. 2 and the amplitude can be written as

$$A_{\tau\to K^\star(K^{\star'})\nu_\tau} = \frac{G_F}{\sqrt{2}} \cdot \bar{u}_{\nu_\tau}\gamma_\alpha u_\tau \cdot g_{\alpha\mu} \cdot |V_{us}| \frac{g_{K^\star}}{2} \int \frac{d^4k}{(2\pi)^4} \frac{tr\ [\gamma_\mu((\hat{k}+\hat{p})+m_u)\gamma_\nu(\hat{k}+m_s]e^\nu_\rho(p_\rho)}{(k^2-m_s^2)((k+p)^2-m_u^2)}, \quad (11)$$

where the m_s is the s- quark mass, and $|V_{us}|$=0.2252 is the Cabibbo - Kobayashi - Maskawa mixing angle.

Using formula (8) for the decay width $\tau \to \rho(\rho')\nu_\tau$, and the following numerical results we obtain

$$\Gamma^{theor}_{\tau\to\rho\nu_\tau} = 2.98 \cdot 10^{-11} \ \ MeV, \tag{12}$$

and

$$\Gamma^{theor}_{\tau\to\rho'\nu_\tau} = 3.306 \cdot 10^{-12} \ \ MeV. \tag{13}$$

The square of the amplitude has an analogous form (7) with the replacement $\rho(\rho') \to K^*(K^{*'})$ and $|V_{ud}| \to |V_{us}|$.

The numerical result

$$\Gamma^{theor}_{\tau\to K^*\nu_\tau} = 2.67 \cdot 10^{-11} \ \ MeV, \tag{14}$$

and

$$\Gamma^{theor}_{\tau\to K^{*'}\nu_\tau} = 1.13 \cdot 10^{-11} \ \ MeV. \tag{15}$$

The experimental data are $\Gamma^{exp}_{\tau\to K^*\nu_\tau} = 3.23 \cdot 10^{-11}$ MeV, and $\Gamma^{exp}_{\tau\to K^{*'}\nu_\tau} = 2.22 \cdot 10^{-11}$ MeV.

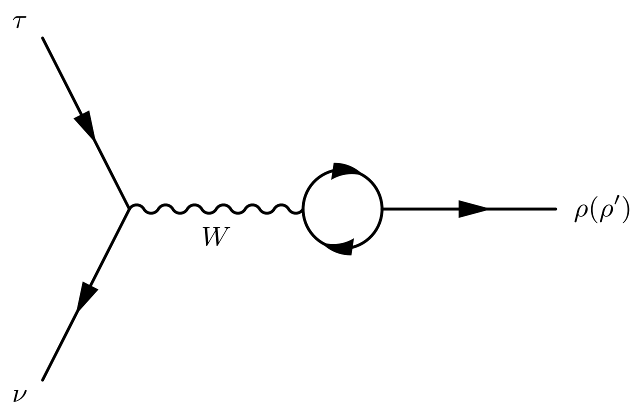

Fig. 1: The Feynman diagram for the decay $\tau \to \rho(\rho')\nu_\tau$.

IV. DISCUSSIONS AND CONCLUSION

The presented here calculations demonsrate that the extended NJL model allows us to describe the decays $\tau \to K^*(892)(K^{*'}(1410))\nu_\tau$ in satistactory agreement with experimental data. Let us emphasize that these results were obtained without using any additional arbitrary parameters. The corresponding estimations for the decay width $\tau \to \rho(770)(\rho'(1450))\nu_\tau$ are also obtained.

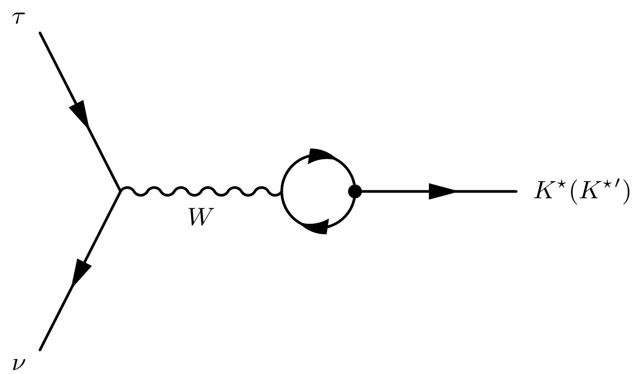

Fig. 2: The Feynman diagram for the decay $\tau \to K^\star(K^{\star'})\nu_\tau$.

The calculations of the amplitude of the decay $\tau \to V\nu_\tau$, where V is the vector meson field, in the one - quark loop approximation in the NJL model take the gradient invariant form $g_{\mu\nu}p^2 - p_\mu p_\nu$. Let us note that both terms for the decay width $\tau \to V\nu_\tau$ in the expression (11) play an important role. Indeed, if we use for the description of both decays $\tau \to V(V')\nu_\tau$, where V' is the first radially excited state, the term $g_{\mu\nu}p^2$, as in [23], then for the decay $\tau \to V'\nu_\tau$ a wrong result will be obtained. On the other hand, it interesting that for all more complicated processes discussed in the Introduction, where vector mesons are the intermediate states, the term $p_\mu p_\nu$ automatically gives zero after multiplication by the vertex describing the vector meson transition to the final product of the corresponding decay. As a result, the diagram containing the term $g_{\mu\nu}p^2$ together with the contact diagram, where W directly goes to the final product through the quark loop, leads to the vector dominant model. It explains the success of the vector dominant model for the description of different τ decays [13–22]. However, in these phenomenological models, for a satisfactory description of experimental data it is necessary to use a set of arbitrary parameters. The extended NJL model allows us to describe the τ decays and e^+e^- processes at low energy without introduction of any additional arbitrary parameters. Using our model in future works we are going to consider more complicated τ decays, in particular, decays with participation of strange particles.

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